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Charts of Complex Function $C\exp(-j\zeta)=[\tanh(T\exp j\tau)]/(T\exp j\tau)$ for Dielectric Measurements

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Charts of the complex function $C\exp(-j\zeta)=[\tanh(T\exp j\tau)]/(T\exp j\tau)$ are given for dielectric measurements based on Roberts and von Hippel's method. The ranges of T and τ to be found with the charts are the values of 0.5 to 3.0 and 58° to 90° respectively. Since the charts are intended to be useful for measurements on media with medium and high dielectric loss, emphasis is laid on the τ range of 58° to 85° which corresponds to that of the dissipation factor from 2.0 to 0.2.

There have been several methods developed for measurements of dielectric constants and loss factors in the region of microwave frequencies[1]. The principle of methods depends on the frequency range concerned, the magnitude of dielectric constant and loss, and some other factors.

In the course of measurements on dielectric properties of polar molecules, it has been necessary to measure the complex dielectric constants of polar liquids in a frequency range of 300 Mc to 6 Gc. In order to obtain the frequency dependence of the dielectric constant at several different frequencies, the short-circuit method by Roberts and von Hippel[2] is most preferable in which the dielectric constant and loss can be evaluated by solving the complex function

$$C\exp(-j\zeta)=\frac{\tanh[T\exp(j\tau)]}{T\exp(j\tau)},$$

where

$$C\exp(-j\zeta)=-j\frac{Z_i}{\beta_0 d},$$

$$T\exp(j\tau)=\gamma_a d,$$

$$\gamma_a=\alpha_a+j\beta_a,$$

Z_i is the normalized input impedance at the face of a dielectric specimen, d the length of the specimen, β_0 the phase constant in the air-filled waveguide, γ_a the propagation constant in the dieletric-filled waveguide, and α_a and β_a the attenuation and phase constants.

The propagation constant is related to the dielectric constant and loss, ϵ' and ϵ'', by the equation for the TE modes

$$\frac{\alpha_d}{\beta_d}=\frac{2\pi}{\lambda_0}\left[\epsilon'-\left(\frac{\lambda_0}{\lambda_c}\right)^2\right]^{1/2}\left\{\frac{1}{2}[(1+D^2)^{1/2}\mp 1]\right\}^{1/2},$$

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where λ_0 is the wavelength in free space, λ_c the cut-off wavelength of the waveguide and $D=\epsilon''/[\epsilon'-(\lambda_0/\lambda_c)^2]$ the dissipation factor in the dielectric-filled waveguide. For the TEM mode in coaxial line the equation simplifies to

$$\begin{matrix}\alpha_d \\ \beta_d\end{matrix}=\frac{2\pi}{\lambda_0}\epsilon'^{1/2}\left\{\frac{1}{2}\left[\left(1+\frac{\epsilon''^2}{\epsilon'^2}\right)^{1/2}\begin{matrix}-\\+\end{matrix}1\right]\right\}^{1/2}.$$

The magnitude and angle, ξ_d and ϕ_d, of the propagation constant γ_d are given by

$$\gamma_d=\xi_d\exp(j\phi_d),$$

$$\left.\begin{aligned}\xi_d&=\frac{2\pi}{\lambda_0}\left[\epsilon'-\left(\frac{\lambda_0}{\lambda_c}\right)^2\right]^{1/2}(1+D^2)^{1/4},\\ \phi_d&=\tan^{-1}\left[\frac{(1+D^2)^{1/2}+1}{(1+D^2)^{1/2}-1}\right]^{1/2},\end{aligned}\right\}\quad\text{for the TE mode}$$

$$\left.\begin{aligned}\xi_d&=\frac{2\pi}{\lambda_0}\epsilon'^{1/2}\left[1+\left(\frac{\epsilon''}{\epsilon'}\right)^2\right]^{1/4},\\ \phi_d&=\tan^{-1}\left\{\frac{[1+(\epsilon''/\epsilon')^2]^{1/2}+1}{[1+(\epsilon''/\epsilon')^2]^{1/2}-1}\right\}.\end{aligned}\right\}\quad\text{for the TEM mode}$$

Using these equations one can determine ϵ' and ϵ'' from the measured quantities, Z_t, d, and λ_0.

Following the original chart by Roberts and von Hippel, more detailed charts for finding T and τ from C and ζ values have been made available which cover the ranges of 0 to 7.0 for T and 70° to 90° for τ[3]. The charts given here range from 0.5 to 3.0 for T and from 58° to 90° for τ. By extending the range of τ value down to 58°, the high dielectric loss to be encountered in the case of dielectric absorption of polar liquids can be easily evaluated from the charts. Thus the τ range of 58° to 85° is emphasized which is that of 2.0 to 0.2 in terms of the dissipation factor D, while the details for τ values greater than 85° are omitted, since the purpose of this work is concerned with measurements of medium and high dielectric loss.

The whole range is covered by 11 charts in which ζ is the ordinate and C or $1/C$ the absissa :

Chart 1. ζ, 0° to 45° ; C, 0.95 to 1.575.
Chart 2. ζ, 0° to 45° ; C, 1.525 to 2.15.
Chart 3. ζ, 0° to 90° ; $1/C$, 0 to 0.625.
Chart 4. ζ, 55° to 100° ; $1/C$, 1.3 to 3.8.
Chart 5. ζ, 40° to 85° ; C, 0.7 to 1.325.
Chart 6. ζ, 40° to 85° ; C, 1.275 to 1.9.
Chart 7. ζ, 80° to 125° ; C, 0 to 0.625.
Chart 8. ζ, 80° to 125° ; C, 0.7 to 1.325.
Chart 9. ζ, 80° to 125° ; C, 1.275 to 1.9.
Chart 10. ζ, 90° to 180° ; C, 0 to 1.225.
Chart 11. ζ, 0° to 180° ; $1/C$, 0 to 1.225.

All calculations were made using KDC-I Digital Computer of Kyoto University. The authors wish to thank Miss K. Tatsumi for drawing charts.

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(2) S. Roberts and A. von Hippel, *J. Appl. Phys.* **17**, 610 (1946).
(3) W. B. Westphal, in A. R. von Hippel, "Dielectric Materials and Applications" p. 86, see Ref. 1.

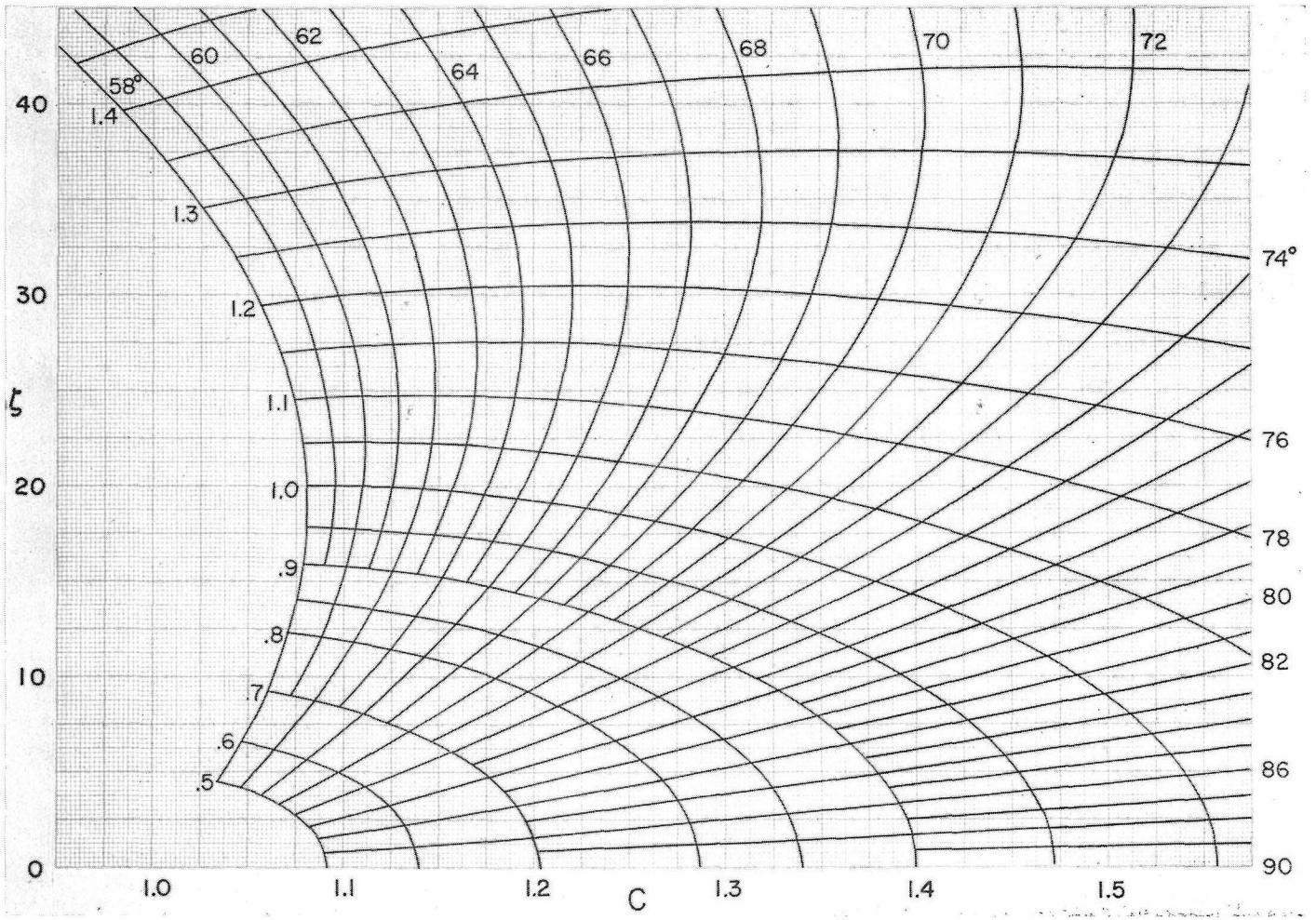

Chart 1. ζ, 0° to 45° ; C, 0.95 to 1.575.

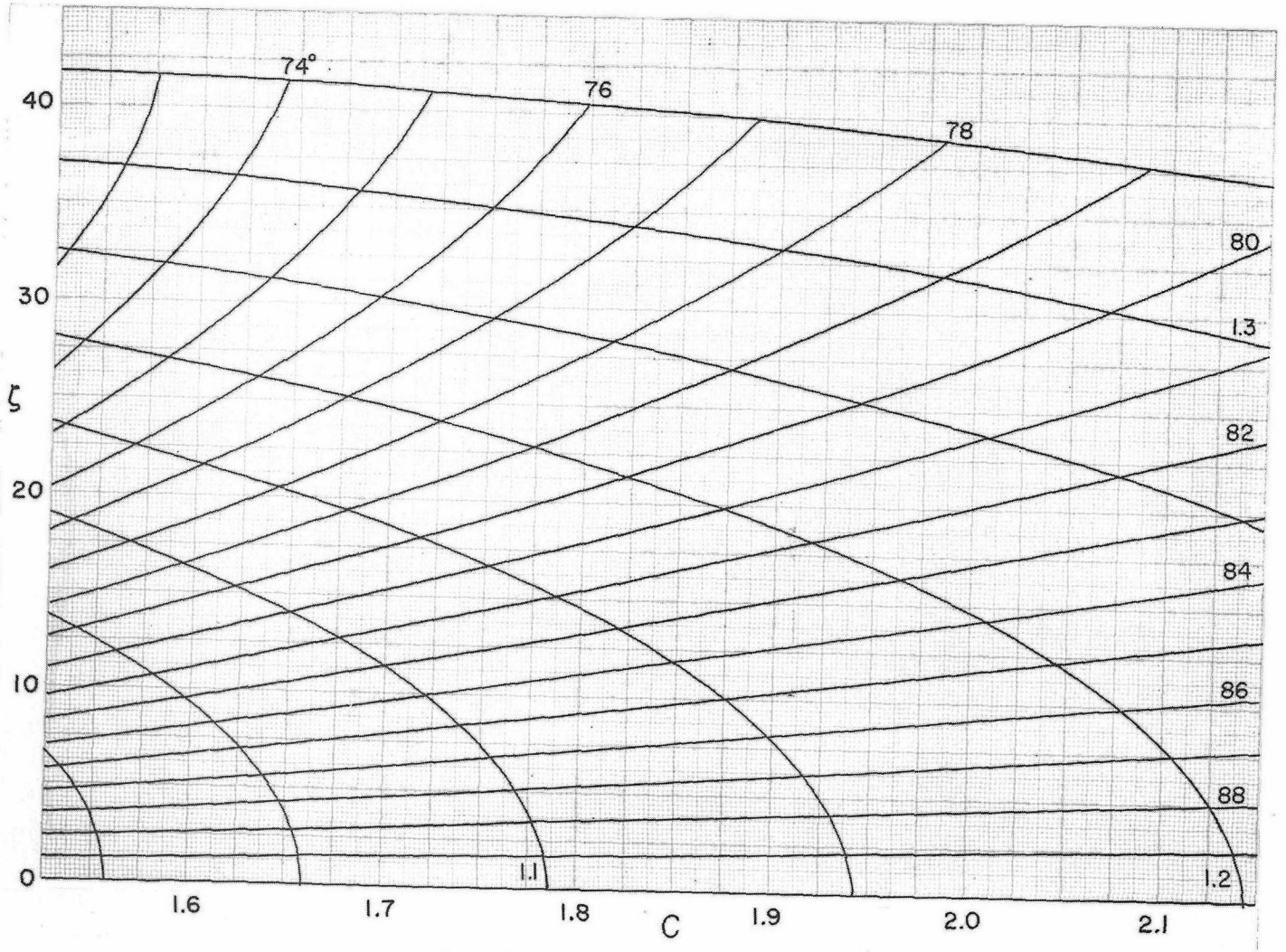

Chart 2. ζ, 0° to 45° ; C, 1.525 to 2.15.

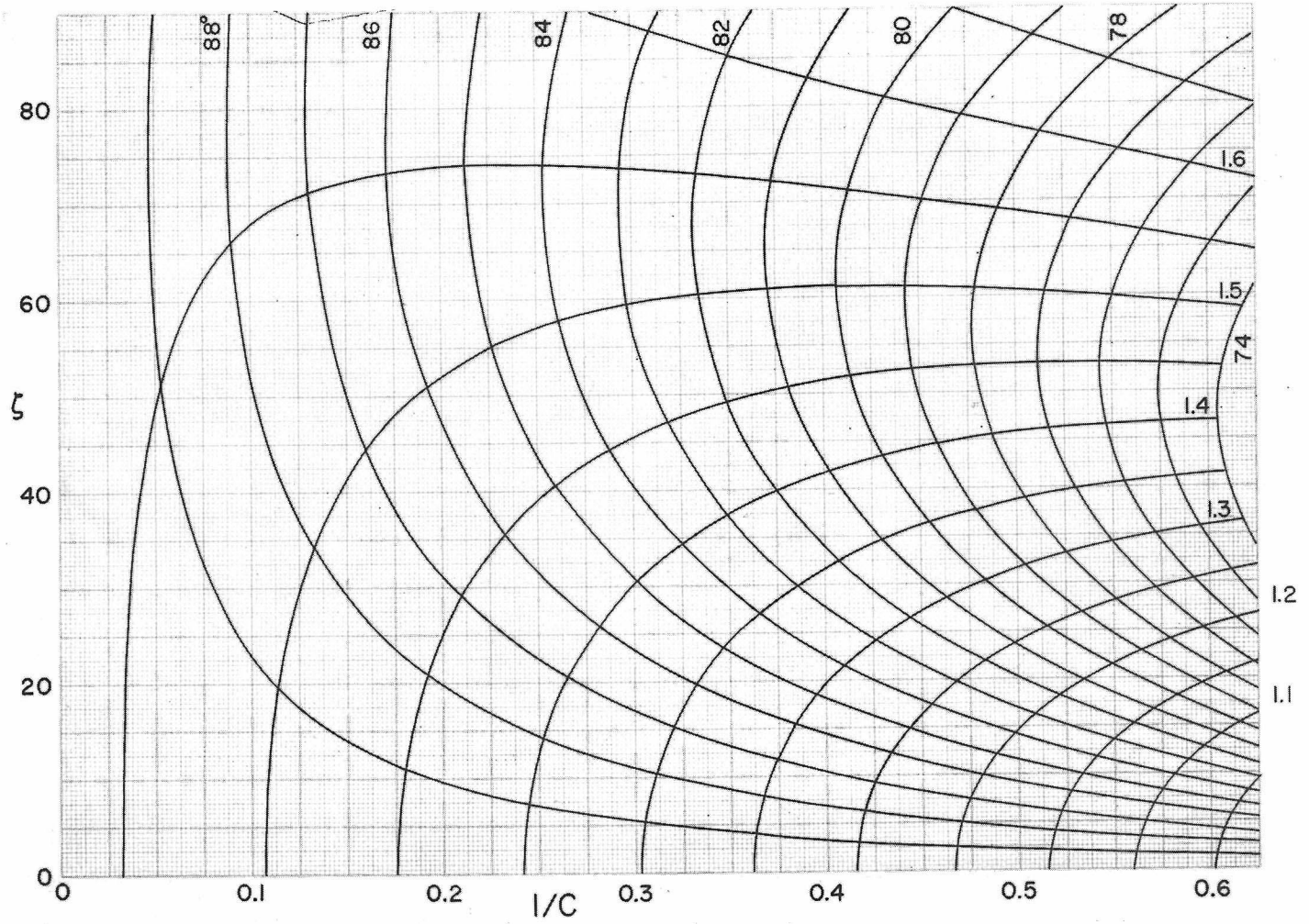

Chart 3. ζ, 0° to 90° ; $1/C$, 0 to 0.625.

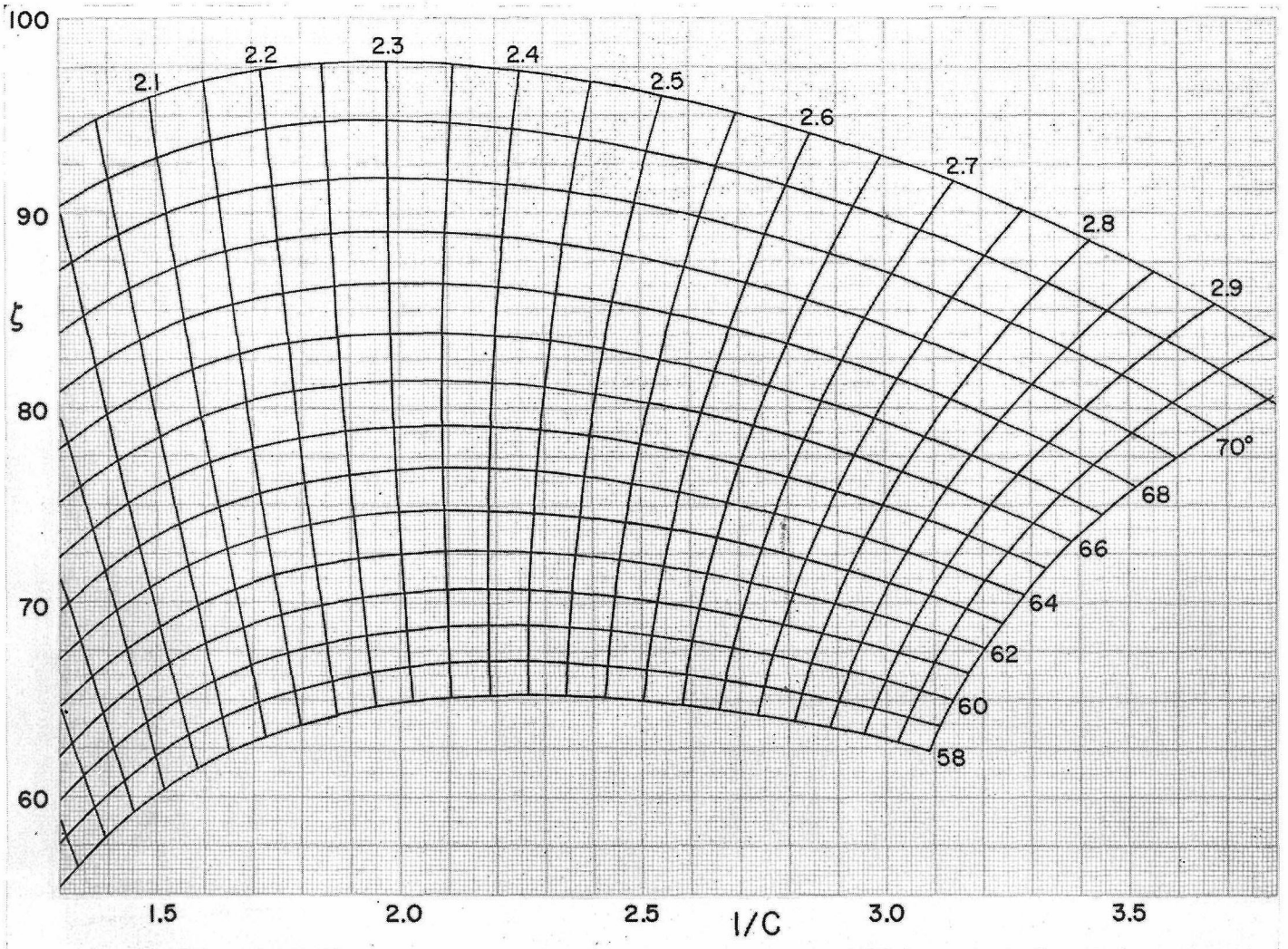

Chart 4. ζ, 55° to 100° ; $1/C$, 1.3 to 3.8.

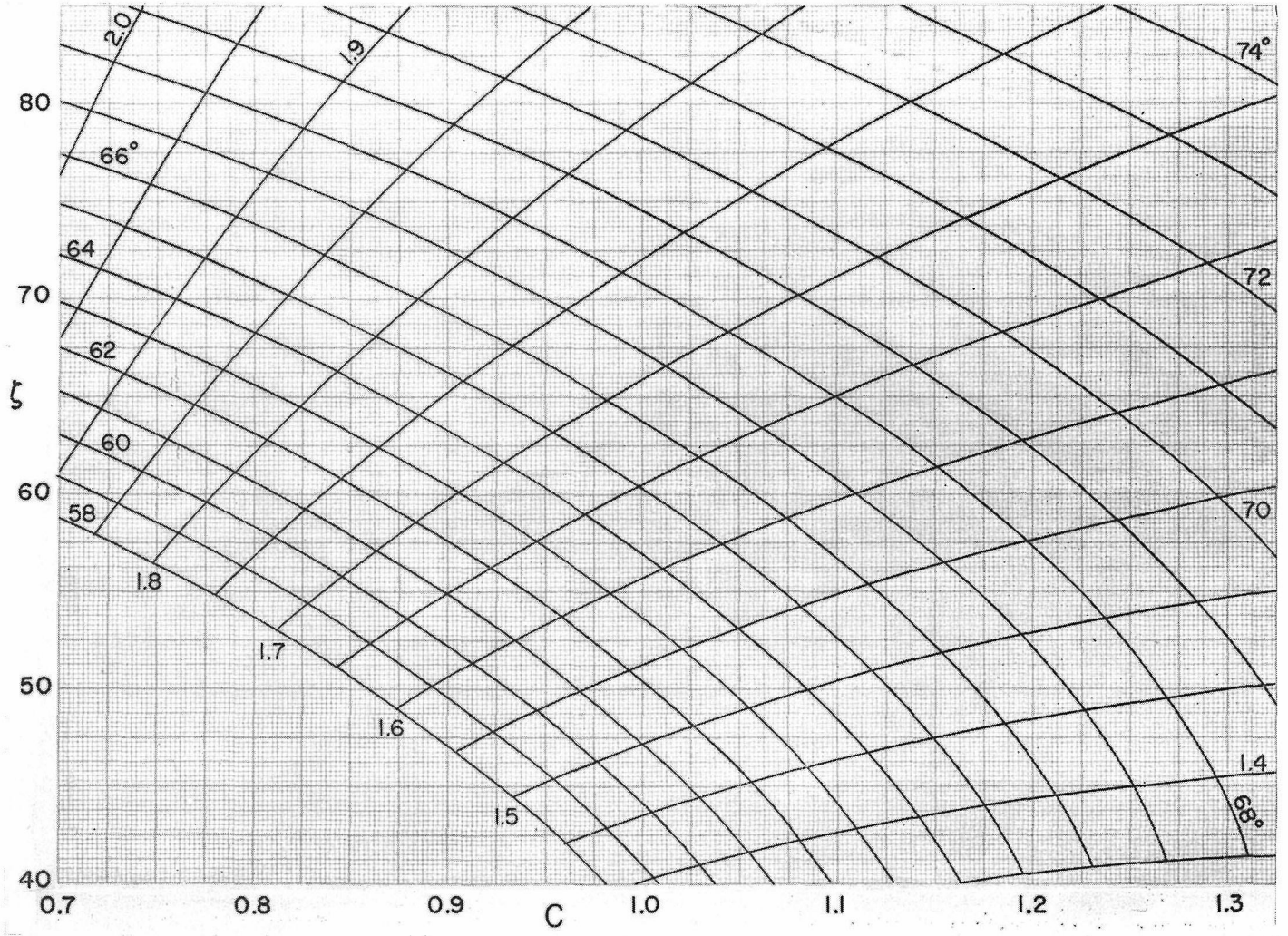

Chart 5. ζ, 40° to 85° ; C, 0.7 to 1.325.

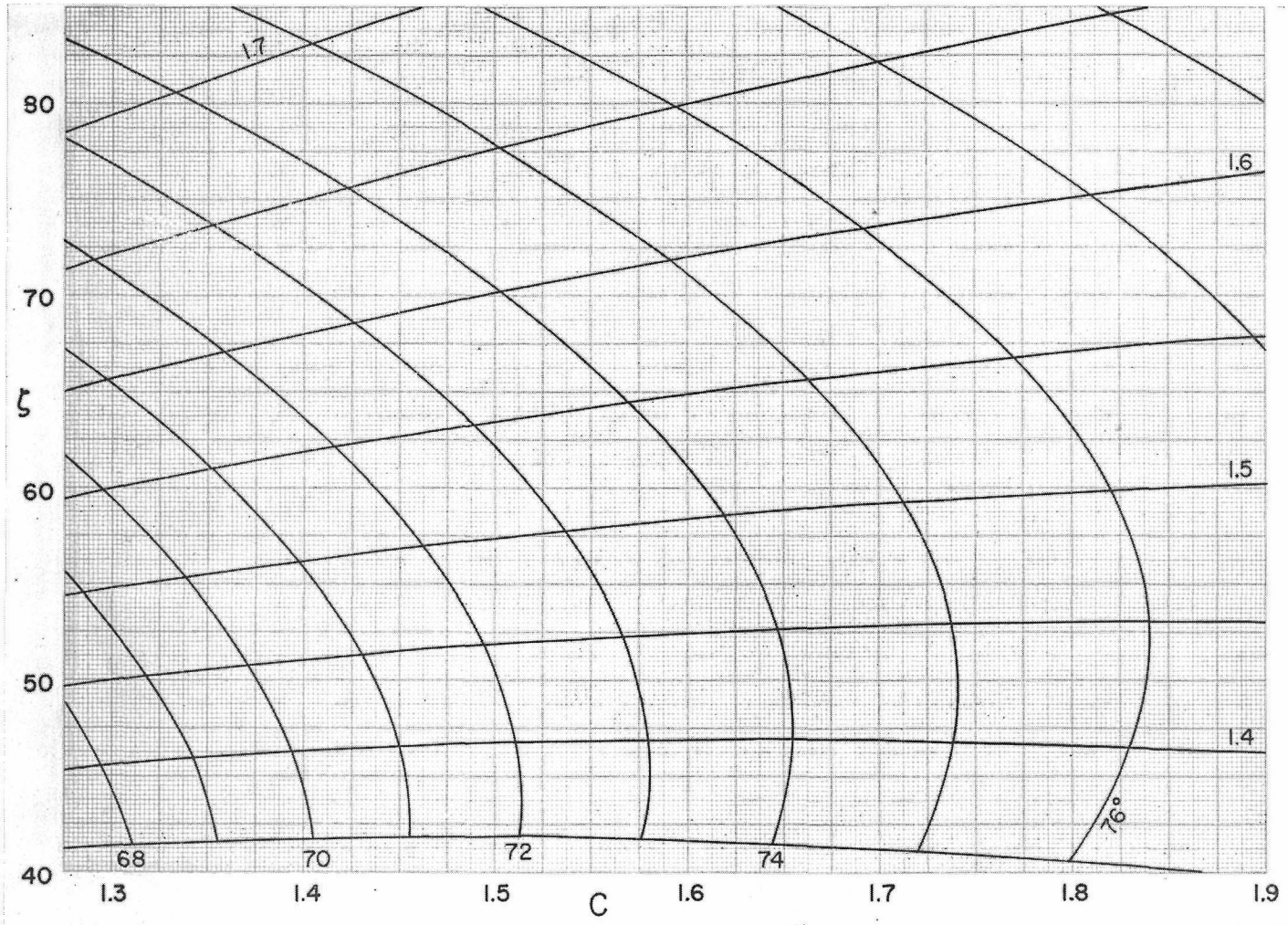

Chart 6. ζ, 40° to 85° ; C, 1.275 to 1.9.

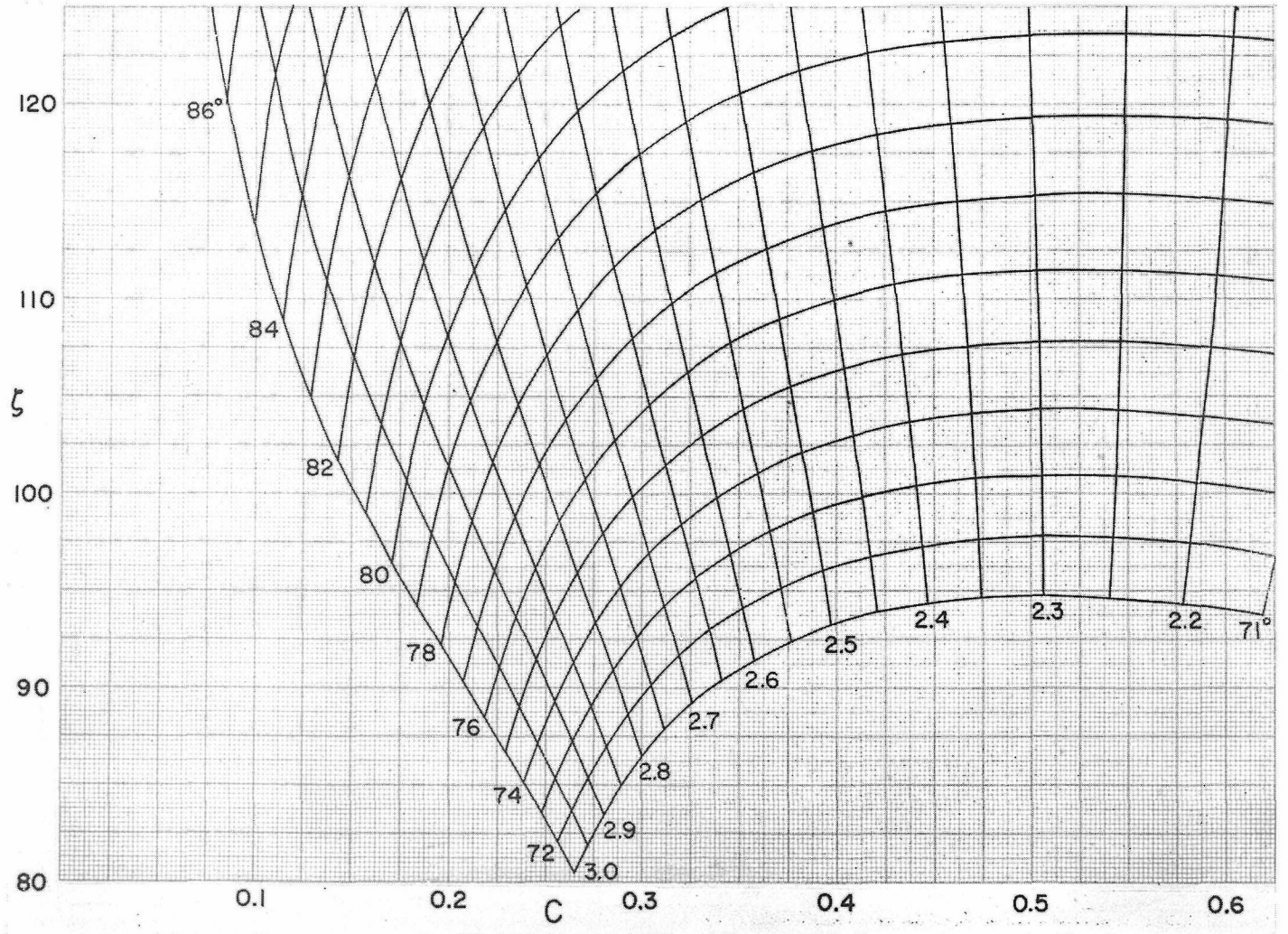

Chart 7. ζ, 80° to 125° ; C, 0 to 0.625.

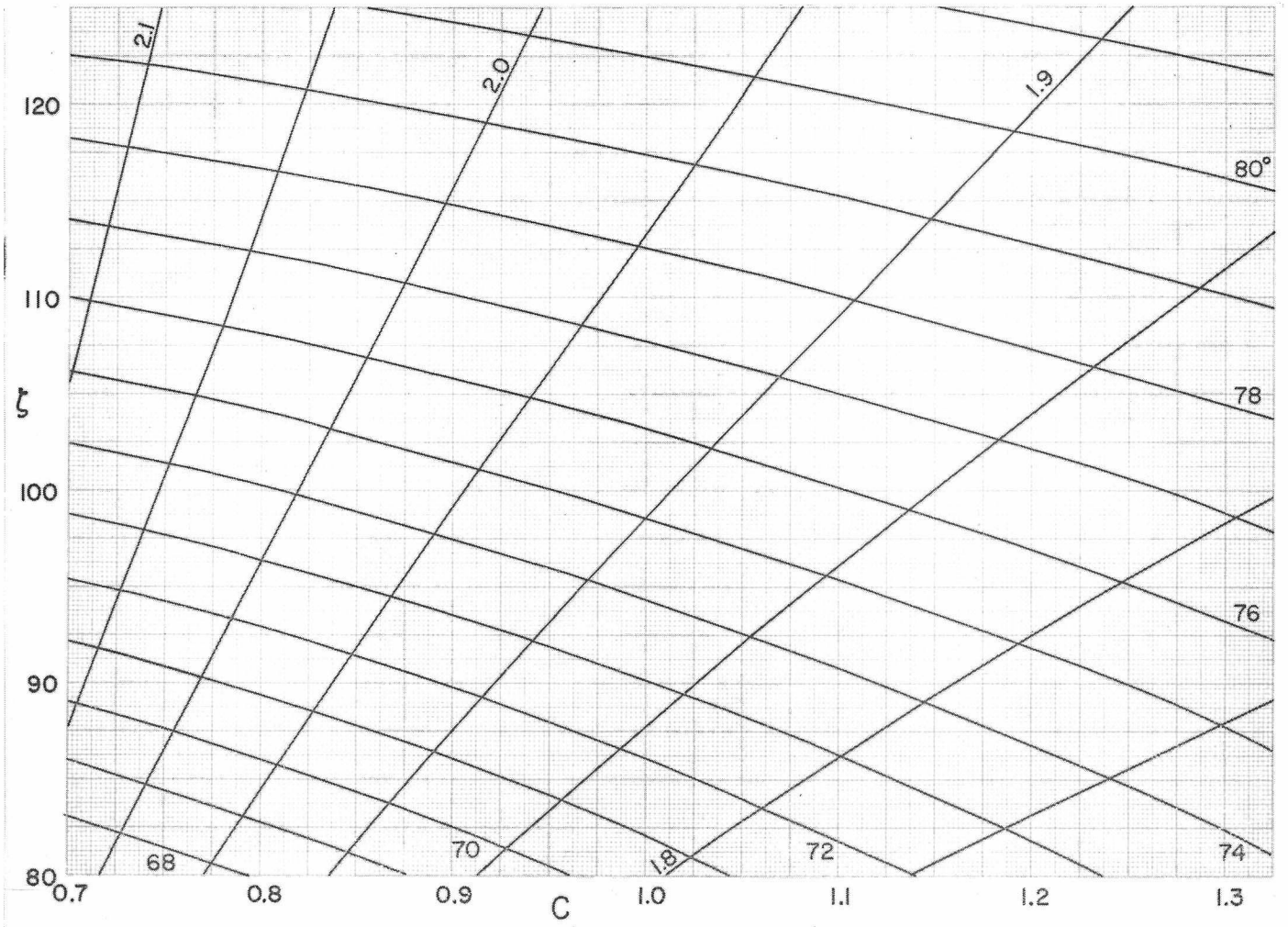

Chart 8. ζ, 80° to 125° ; C, 0.7 to 1.325.

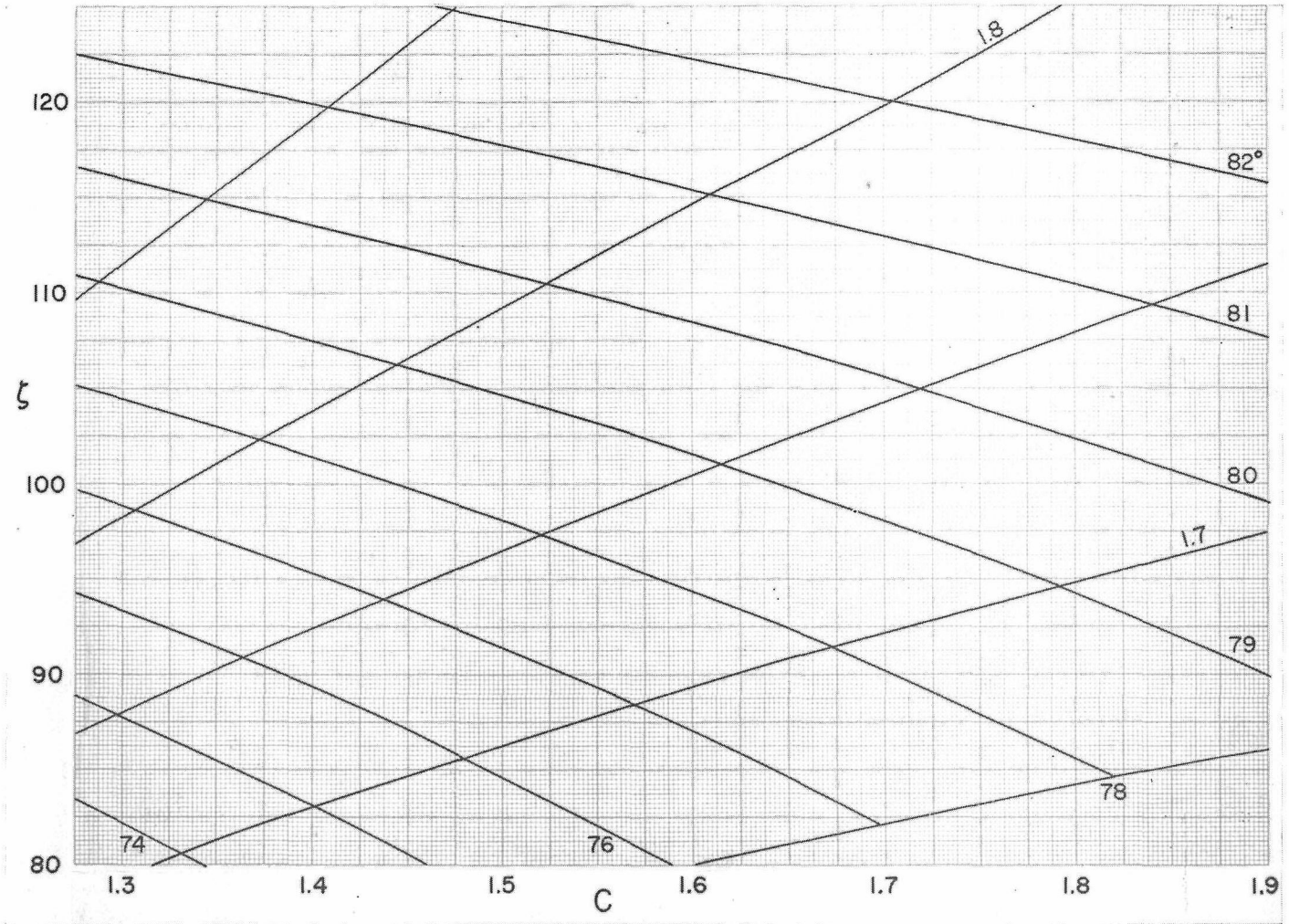

Chart 9. ζ, 80° to 125° ; C, 1.275 to 1.9.

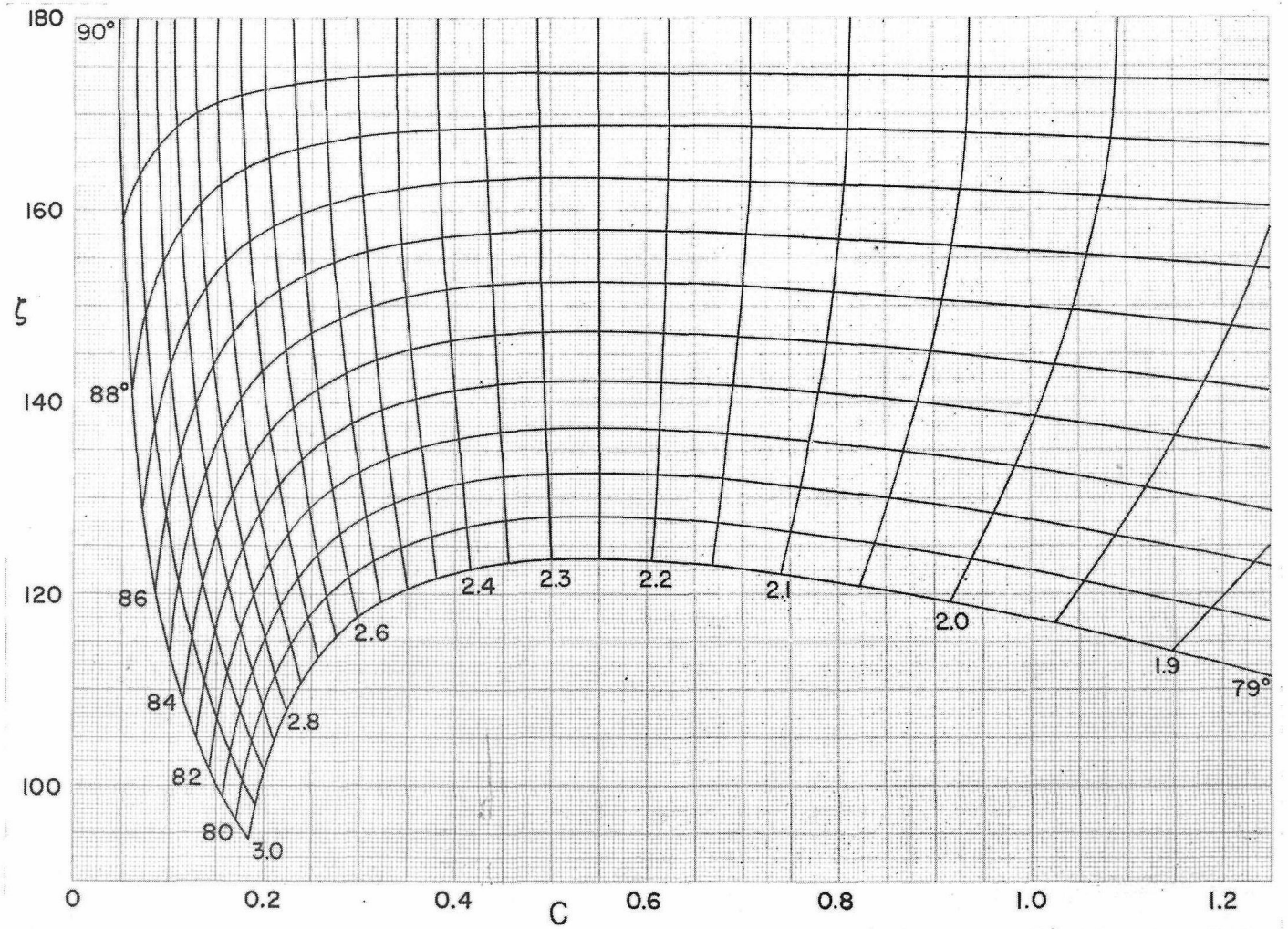

Chart 10. ζ, 90° to 180° ; C, 0 to 1.225.

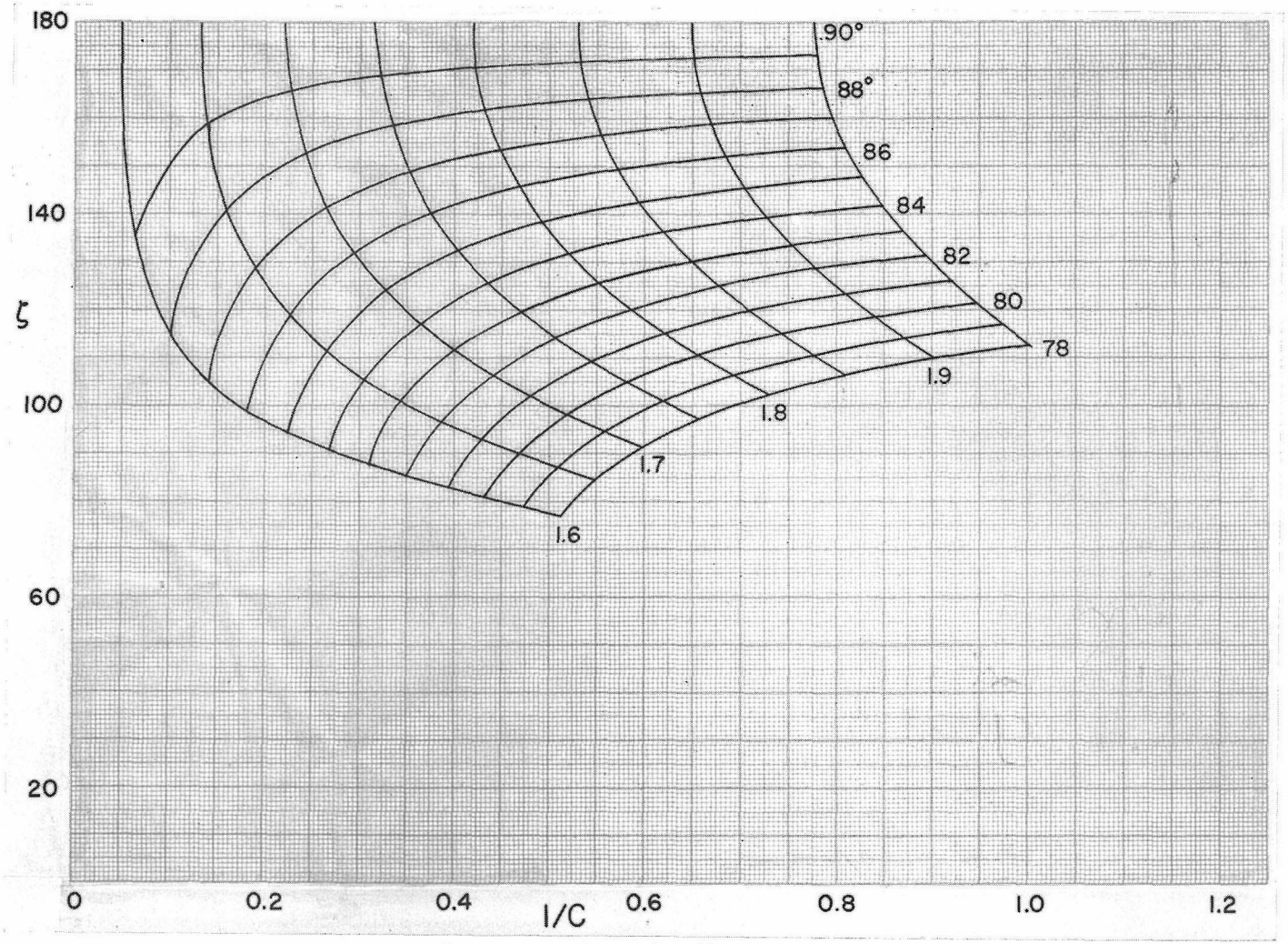

Chart 11. ζ, 0° to 180° ; $1/C$, 0 to 1.225.